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Hiding an extra dimension

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Abstract: We propose a new geometry and/or topology of a single extra dimension whose Kaluza-Klein excitations do appear at much higher scale than the inverse of the length/volume. For a single extra dimenion with volume $N\pi\rho$ which is made of N intervals with size $\pi\rho$ attached at one point, Kaluza-Klein excitations can appear at $1/\rho$ rather than $1/N\rho$ which can hide the signal of the extra dimenion sufficiently for large N. The geometry considered here can be thought of a world volume theory of self intersecting branes or an effective description of complicated higher dimensional geometry such as Calabi-Yau with genus or multi-throat configurations. This opens a wide new domain of possible compactifications which deserves a serious investigation.

Keywords: Field Theories in Higher Dimensions, Flux compactifications, Intersecting branes models, Large Extra Dimensions.

JHEP01(2006)090

Contents

1. Introduction

Unification of gauge and gravitational interactions is one of the most important paradigm in particle physics and it has guided theoretical physics when the experiments did not follow theory. Three gauge couplings are believed to be unified at very high energy so called grand unification scale (GUT scale). In the standard model it works within 10 to 20 percent errors and in the minimal supersymmetric extensions of it, the unification works a lot better (within a few percent errors). Thus it seems to provide a strong hint for what is new physics at TeV scale or higher. In order to unify gauge interactions with gravity, first we should understand why the electroweak scale is so lower compared to the Planck scale at which gravitational interactions become of order one similar strength to the gauge interactions. Supersymmetry broken at TeV is regarded as the most popular solution to this problem.

However, we can address the question in a different way. Why is gravity so weak? Effective gravitational interaction at given energy scale is E^2/M_{Planck}^2 and is extremely tiny compared to order one gauge interactions. This question brought entirely new solutions to the problem of disparity between gravity and gauge interactions in terms of extra dimensions. Large extra dimension [1, 2] explains the weakness of gravity in terms of large volume of extra dimenions only gravity feels. Warped extra dimension (a slice of AdS_5)

JHEP01(2006)090

proposed by Randall and Sundrum [3] naturally provides TeV brane at which the natural scale is just TeV due to an exponential warp factor along the extra dimension. Graviton zero mode wave function is not flat in AdS_5 but is localized at Planck brane. Thus TeV brane matter feels only the tail of graviton zero mode and weakness of gravity is naturally explained even with a small (order one) size of the extra dimension.

Flat extra dimension with size smaller than 0.1mm is consistent with the current experimental limit [4] as long as gauge interactions are confined on the brane and only gravity feels it. Submillimeter extra dimensions make gravity be strong at TeV if there are two extra dimensions which is just the limit from precision gravity experiment. Although it provides the most interesting possibility, there comes a strong constraint from astrophysics/cosmology. From the supernovae and neutron stars we would expect more gamma rays from decays of massive Kaluza-Klein gravitons whose mass is below the temperature of the supernovae core, 30 MeV. This puts the most stringent bound on large extra dimensions [5]. Single extra dimension gives too light massive graviton which is already inconsistent with the experimental fact if we force the scale of quantum gravity at around TeV. For two extra dimensions, the bound pushes the scale of quantum gravity beyond 1000 TeV and we can not relate it to the weak scale any longer. In this paper we suggest a setup in which the lightest Kaluza-Klein graviton is heavy enough and can be consistent with the experimental bounds. In this setup the N-fold degeneracy with sufficiently large N provides a rapid change of the gravitational interactions such that gravity can be of order one at TeV.

String theory is usually defined in 10/11 dimensions and 6/7 extra dimensions should be curled up and be hidden to be consistent with the fact that we live in 3+1 noncompact spacetime. The most popular scenario assumes Calabi-Yau space as the compactification manifold to yield 4D N=1 supersymmetry [6]. Recently compactification with various flux has been intensively studied as it provides the stabilization of most string theory moduli which otherwise would remain massless [7 − 11]. Flux compactification also generates throat geometry in Calabi-Yau and the long throat physics is well described in terms of effective 5 dimensional theory. Full 10 dimensional physics appears only at very high energy scale near the string scale and the low energy excitations are just the Kaluza-Klein states of Randall-Sundrum like setup. It is then natural to imagine that there would be many throats in Calabi-Yau space and we can ask what the theory looks like if Calabi-Yau has multi-throat geometry. In this case we have a clear distinction between scales of Kaluza-Klein excitations and light modes appear only at around infrared(IR) branes. There are many physical questions that can be addressed without knowing full 10 dimensional spectrum. Therefore it would be interesting to see what the spectrum will look like for the multi-throat geometry. The essential property of multi-throat geometry is kept when we replace each throat by RS geometry which just include single extra coordinate [12 − 14]. [1] Then the bulk region corresponds to the ultraviolet (UV) brane. As all the throats are connected to the bulk, several IR branes are linked to the UV brane through the slice of AdS_5. This setup is exactly the one we will study here.

[1]Recent studies are in [15, 16].

Once we have a situation where the extra coordinate is just one but has a several branch starting from the UV brane, we can generalize it to the flat space. The junction of extra space is nothing to do with the curvature of each AdS_5 and we can attach several different AdS_5 slice with different curvatures at the same time. Therefore, it is natural to imagine the flat limit of the same configuration. At least we can define a consistent field theory on the flat limit of the multi-throat effective theory and can study the theory on it. How to get such a geometry from Calabi-Yau or other compactification is an independent question and we will not address it here. One obvious example is the torus with a genus one. When one cycle wrapping the genus is much larger than the other cycle, we can approximate the geometry as one dimensional ring at low energy scale. The excitation associated to the other cycle will appear only at very high energy scale and will be irrelevant to the physics below the inverse scale of the other cycle. We can find an effective 5 dimensional description of multi-geni Calabi-Yau in a similar way.

In this paper we will analyze the spectrum of the fields living in a single extra dimension discussed above. After a brief discussion on how to get such an extra dimension, we use deconstruction with a few sites for the analysis. We also study the phenomenology with spectrum obtained by deconstruction technique. Then we discuss the actual analysis in field theory. Finally we conclude with a few remarks.

2. Brane intersection of its own

As long as gauge interactions are concerned, the best way to obtain the flat space limit of multi-throat geometry is the brane intersection of its own. We consider a setup in which a brane bends and finally intersects by itself. The simplest possibility is to have figure eight(8). We can continue the process so that many rings intersect at a single point. Perhaps the simplest one is to fold the ring in such a way there would be an interval. The final setup would be the gathering of many intervals with one common point. Suppose that the individual interval has a finite length ρ and there are N such intervals. The total length is then $N\rho$. Any gauge theory living on this configuration would have a suppression $1/(N\rho)$ in its 4D gauge coupling. Now the question is the scale of Kaluza-Klein excitations.

Thus we consider these configurations. To see the new feature clearly, we take the deconstruction [17, 18] as our analysis tool.

2.1 Deconstruction

If we do the analysis for the circle moose diagram, we would obtain the eigenvalues

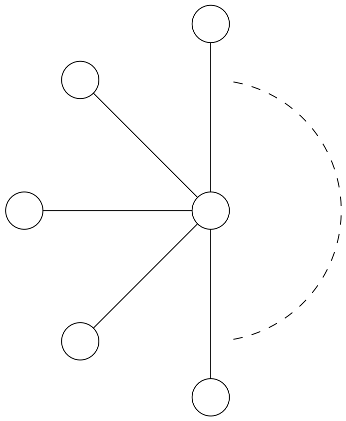
$$M_n^2 = \left(\frac{2}{a}\right)^2 \sin^2\left(\frac{na}{2R}\right), \quad \frac{-N}{2} < n \leq \frac{N}{2}$$

where $a = \frac{1}{g\langle\Phi\rangle}$ and $R = Na$. For $N \gg 1$ and $n \ll N$, the expression is well approximated to be

$$M_n^2 = \left(\frac{n}{R}\right)^2.$$

2.1.1 N-octopus

First of all, suppose there is a center point at which several intervals are connected. We call it 'octopus' diagram although the legs need not be eight. Let the legs be N. Each leg has one end adjacent to the head of the octopus (the center). The boundary condition would determine the eigen modes along the extra dimension but it would be easier to see it from a simplified deconstruction setup.



Let us consider a gauge theory on it. There is a gauge boson A^0_μ which is at the head and each leg connects A^0_μ to A^i_μ where $i = 1, \cdots, N$. If the scalar fields linking two sites get VEVs, the corresponding gauge bosons become massive. The link field Φ_i is bi-fundamental under the gauge group G^0 and G^i. The mass matrix for $N+1$ gauge bosons is

$$M^2 = \frac{1}{a^2}\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 1 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 1 & \cdots & 0 & -1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ -1 & -1 & -1 & \cdots & -1 & N \end{pmatrix} \tag{2.1}$$

where $a = \frac{1}{g\langle\Phi\rangle}$ and the $N+1$th column and row correspond to A^0_μ. There are $N+1$ eigenstates. The characteristic equation can be easily derived for $\hat{M}^2 = a^2 M^2$.

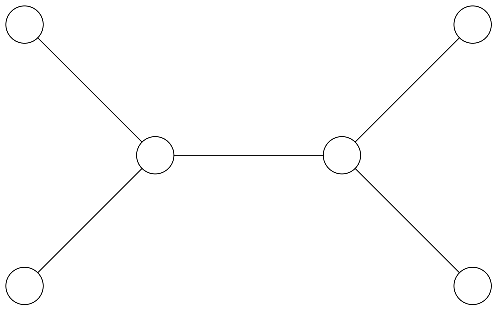
$$\det(\hat{M}^2 - \lambda I) = \lambda(1-\lambda)^{N-1}\{\lambda - (N+1)\} \tag{2.2}$$

There is a zero mode $\lambda = 0$ with the eigenvector $v_0 = \frac{1}{\sqrt{N+1}}(1,1,1,\cdots,1)$. The lightest Kaluza-Klein states are degenerate. There are $N-1$ states with mass $(\frac{1}{a})^2$. The eigenvectors should be orthogonal to the zero mode and its $N+1$th component is zero. Thus $v_1 = \frac{1}{\sqrt{2}}(1,-1,0,\cdots,0,0,0,\cdots,0)$, $v_2 = \frac{1}{\sqrt{6}}(1,1,-2,\cdots,0,0,0,\cdots,0)$ and $v_i = \frac{1}{\sqrt{i(i+1)}}(1,1,1,\cdots,1,-i,0,\cdots,0)$ where $i = 1,\cdots,N-1$. (The final one with $i = N$ is not linearly independent if there are vectors from $i = 1$ to $i = N - 1$.) The last one has the eigenvalue $\frac{(N+1)}{a^2}$ and the eigenvector is $v_N = \frac{1}{\sqrt{N(N+1)}}(1, 1, \cdots, -N)$.

The deconstruction of the octopus with N legs can be easily generalized to include higher excitations of each leg by adding more sites between the site 0 and i. The octopus has two distance scales. One is the size of each leg ρ which is just the lattice size in the above example $\rho = a$. The other is the total volume of the extra dimension which is simply N times ρ. ($R = N\rho$). You might guess that the lowest excitation will appear at a scale $1/R$ but it turns out that it appears only at $1/\rho = N/R$. It is an interesting example in which the volume suppression can be large and at the same time the Kaluza-Klein excitations associated with it can be very heavy.[2]

2.1.2 Two centers

Let us consider the second example with two centers.



It is straightforward to generalize the setup.

$$M^2 = \frac{1}{a^2}\begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \tag{2.3}$$

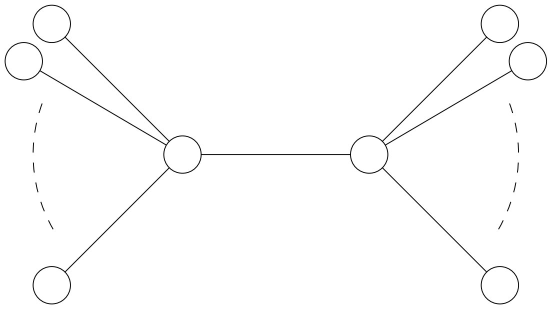
We can list the eigenvalues and the eigenstates for $\hat{M}^2 = a^2 M^2$ up to normalization.

$$\begin{aligned} \lambda = 0 \qquad & (1, 1, 1, 1, 1, 1) \\ \lambda = \frac{5 - \sqrt{17}}{2} \qquad & \left(1, 1, \frac{\sqrt{17} - 3}{2}, -\left(\frac{\sqrt{17} - 3}{2}\right), -1, -1\right) \\ \lambda = 1 \qquad & (1, -1, 0, 0, 0, 0) \end{aligned}$$

[2] With two or more extra dimensions, distinct KK modes appear if we consider compact hyperbolic extra dimensions [19].

$$
\begin{aligned}
\lambda = 1 \qquad & (0,0,0,0,1,-1) \\
\lambda = 3 \qquad & (1,1,-2,-2,1,1) \\
\lambda = \frac{5+\sqrt{17}}{2} \qquad & \left(1,1,-\left(\frac{3+\sqrt{17}}{2}\right),\frac{3+\sqrt{17}}{2},-1,-1\right)
\end{aligned}
$$

2.1.3 Two centers with 2N legs



$$
M^2 = \frac{1}{a^2}\begin{pmatrix}
1 & 0 & \cdots & 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & -1 & 0 & 0 & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\
-1 & -1 & \cdots & -1 & N+1 & -1 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & -1 & N+1 & -1 & \cdots & -1 & -1 \\
0 & 0 & \cdots & 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 0 & 0 & -1 & 0 & \cdots & 1 & 0 \\
0 & 0 & \cdots & 0 & 0 & -1 & 0 & \cdots & 0 & 1
\end{pmatrix} \tag{2.4}
$$

We can list the eigenvalues and the eigenstates for $\hat{M}^2 = a^2 M^2$ up to normalization. For large N ($N \gg 1$), the expression can be approximated as follows.

$$
\begin{aligned}
\lambda = 0 \qquad & (1,1,\cdots,1,1,1,1,\cdots,1,1) \\
\lambda = \frac{2}{N} \qquad & \left(1,1,\cdots,1,1-\frac{2}{N},-1+\frac{2}{N},-1,\cdots,-1,-1\right) \\
\lambda = 1 \qquad & (1,-1,\cdots,0,0,0,0,\cdots,0,0) \\
& \cdots \\
& (1,1,\cdots,-N+1,0,0,0,\cdots,0,0) \\
& (\mathrm{N}-1) \\
\lambda = 1 \qquad & (0,0,\cdots,0,0,0,1,-1,\cdots,0) \\
& \cdots
\end{aligned}
$$

$$(0,0,\cdots,0,0,0,1,1,\cdots,-(N-1))$$

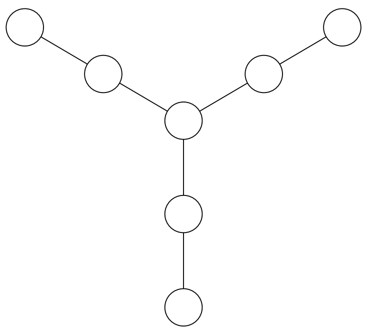
$$(\mathrm{N}-1)$$

$$\lambda = N+1 \qquad (1,1,\cdots,1,-N,-N,1,\cdots,1,1)$$

$$\lambda = N+3-\frac{2}{N} \quad \left(1,1,\cdots,1,-N-2+\frac{2}{N},N+2-\frac{2}{N},-1,\cdots,-1,-1\right)$$

The presence of light modes $\lambda = \frac{2}{N}$ is the most striking aspect of two centers model. When there is a unique center, the lightest excitation started from 1. Now it starts from $\frac{2}{N}$ which is very light for $N \gg 1$. Interpretation of the result is simple. If we disconnect the middle line connecting two centers, we end up with two 'N-Octopus' and each one has a zero mode. If we connect two centers with a new line, it becomes a coupled system which mimics two ground state problem in quantum mechanics. If there is a small mixing, the true ground state is an even combination of two ground states and there is an excited state which is an odd combination of the two ground states. If the mixing vanishes, there are twofold degenerate ground state. Here the middle line plays a role of the mixing between two states and we get one zero mode (even combination of each N-octopus zero mode) and one light mode (odd combination of each one). [3]

2.2 3 legs with multiple sites



$$M^2 = \frac{1}{a^2}\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 & 3 \end{pmatrix} \qquad (2.5)$$

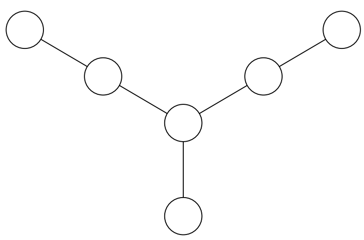
[3]The author thanks R. Rattazzi for this simple interpretation.

We can list the eigenvalues and the eigenstates for $\hat{M}^2 = a^2 M^2$ up to normalization.

$$
\begin{array}{ll}
\lambda = 0 & (1,1,1,1,1,1,1) \\
\lambda = \dfrac{3-\sqrt{5}}{2} & \left(1, \dfrac{\sqrt{5}-1}{2}, -1, \dfrac{-\sqrt{5}+1}{2}, 0, 0, 0\right) \\
 & \left(1, \dfrac{\sqrt{5}-1}{2}, 1, \dfrac{\sqrt{5}-1}{2}, -2, -\sqrt{5}+1, 0\right) \\
\lambda = \dfrac{3+\sqrt{5}}{2} & \left(1, \dfrac{-\sqrt{5}-1}{2}, -1, \dfrac{\sqrt{5}+1}{2}, 0, 0, 0\right) \\
 & \left(1, \dfrac{-\sqrt{5}-1}{2}, 1, \dfrac{-\sqrt{5}-1}{2}, -2, \sqrt{5}+1, 0\right) \\
\lambda = 3-\sqrt{2} & \left(1, -2+\sqrt{2}, 1, -2+\sqrt{2}, 1, -2+\sqrt{2}, 3-3\sqrt{2}\right) \\
\lambda = 3+\sqrt{2} & (1, -2-\sqrt{2}, 1, -2-\sqrt{2}, 1, -2-\sqrt{2}, 3+3\sqrt{2})
\end{array}
$$

The result shows that the addition of nodes provides more modes which are heavier than the energy scale corresponding to the inverse of each leg. One clear thing is that there is no mode whose scale is about $1/(6a)^2$.

2.3 3 legs with multiple sites (different lengths)



$$
M^2 = \frac{1}{a^2} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 0 & -1 & 3 \end{pmatrix}
$$

We can list the eigenvalues and the eigenstates for $\hat{M}^2 = a^2 M^2$ up to normalization.

$$\begin{aligned}
\lambda &= 0 & &(1,1,1,1,1,1) \\
\lambda &= \frac{3-\sqrt{5}}{2} & &\left(0,1,\frac{\sqrt{5}-1}{2},-1,\frac{-\sqrt{5}+1}{2},0\right) \\
\lambda &= \frac{5-\sqrt{13}}{2} & &\left(1,-\frac{1}{2},\frac{3-\sqrt{13}}{4},-\frac{1}{2},\frac{3-\sqrt{13}}{4},\frac{\sqrt{13}-3}{4}\right) \\
\lambda &= 2 & &(1,1,-1,1,-1,-1) \\
\lambda &= \frac{3+\sqrt{5}}{2} & &\left(0,1,\frac{-\sqrt{5}-1}{2},-1,\frac{\sqrt{5}+1}{2},0\right) \\
\lambda &= \frac{5+\sqrt{13}}{2} & &\left(1,-\frac{1}{2},\frac{3+\sqrt{13}}{4},-\frac{1}{2},\frac{3+\sqrt{13}}{4},\frac{-3-\sqrt{13}}{2}\right)
\end{aligned}$$

2.4 Large extra dimensions

Although we can not apply the deconstructed result directly to gravity, the field theory analysis would give the same result. Now the Kaluza-Klein states appear at very high scales.

Deviation of Newtonian potential can be understood in terms of 4 dimensional effective theory. With massless graviton only, the potential between two test particles with mass m_1 and m_2 separated by distance r is

$$\frac{V}{G_N m_1 m_2} = \frac{1}{r}. \tag{2.6}$$

If we consider 5 dimensional theory compactified on a circle with radius R, we have extra massive states with $M_n = \frac{n}{R}$ for $n = 1, 2, \cdots$. They also mediate gravitational interactions by Yukawa potentials

$$\begin{aligned}
\frac{\delta V}{G_N m_1 m_2} &= \sum_{n=1}^{\infty} \frac{e^{-M_n r}}{r} \\
&= \frac{e^{-\frac{r}{R}}}{r(1-e^{-\frac{r}{R}})}.
\end{aligned} \tag{2.7}$$

If $r \gg R$, $\frac{\delta V}{V} \ll 1$ and we just have 4 dimensional gravity. However, if $r \ll R$, $\frac{\delta V}{V} \gg 1$ and $\frac{\delta V}{G_N m_1 m_2} \simeq \frac{R}{r^2}$ and

$$V \simeq G_N^{(5)} \frac{m_1 m_2}{r^2}, \tag{2.8}$$

which produce 5 dimensional gravitational potential ($G_N^{(5)} = RG_N$, 5 dimensional Newton's constant).

We can do the same thing for higher dimensions but now the exact summation formula is not available. When $r \ll R$, we can approximate the summation with integrals

$$\frac{\delta V}{G_N m_1 m_2} = \sum_{n_1, n_2, \cdots, n_{D-4}=1}^{\infty} \frac{e^{-M_n r}}{r}$$

$$= \int_{n=1}^{\infty} dn C_{D-4} n^{D-5} \frac{e^{-\frac{nr}{R}}}{r}$$
$$= C'_{D-4} \frac{R^{D-4}}{r^{D-3}}$$

where $M_n = n/R$ with $n = \sqrt{n_1^2 + n_2^2 + \cdots n_{D-4}^2}$ for the isotropic compactification ($R_1 = R_2 = \cdots = R_{D_4} = R$). C_{D-4} is the solid angle of $D-4$ dimension.

Now let us consider the 'N-Octopus' configuration. If we consider the setup in which N equal length intervals with size $\pi\rho$ attached at a single point (total length $= \pi R = \pi N \rho$), the Kaluza-Klein spectrum comes as N degenerate states at $M_n = n/\rho$. In this case Newtonian potential is modified by

$$\frac{\delta V}{G_N m_1 m_2} = \sum_{n=1}^{\infty} N \frac{e^{-M_n r}}{r} = N \frac{e^{-\frac{r}{\rho}}}{r(1 - e^{-\frac{r}{\rho}})}, \tag{2.9}$$

and when $r \ll \rho$, we have

$$\frac{\delta V}{G_N m_1 m_2} = \frac{N\rho}{r^2} = \frac{R}{r^2}. \tag{2.10}$$

Therefore, we can conclude that it just reproduces 5 dimensional gravity when $r \ll \rho \ll R = N\rho$. Note the relation between R and ρ. If $N \gg 1$, there is a huge difference between the scales at which the gravity is modified and the scale that enters in the modified potential. The correction from massive gravitons become of order one if $\frac{\delta V}{V} \sim \mathcal{O}(1)$ and it is when the critical radius $r_c \simeq \rho \log N$ which is not so much different from ρ. The scale entering in 5D potential is $R = N\rho$ which is much larger distance scale than ρ or $\rho \log N$. In this way we can simple imagine 5D flat extra dimensional model in which the fundamental scale is around TeV while avoiding the phenomenological constrants from the experiments.

We can choose N large enough to make a single extra dimension scenario be consistent with the current experimental bound. For $r_c \simeq 0.1mm$ ($1/r_c \simeq 10^{-3}$ eV), if $1/\rho = 10^{-1}$ or 10^{-2} eV and $N = 10^{16}$ or 10^{17}, we can explain the weak scale quantum gravity with only single extra dimension.

On the other hand, the most stringent bound on the extra dimension comes from supernovae and neutron stars. This bound is not applicable if KK mass is heavier than 100 MeV. Thus for $1/\rho = 100$ MeV and $N = 10^{25}$, we start to see the fifth dimension when $1/r_c \sim 1$ MeV and the gravity becomes strong at TeV. Octopus configuration with large N can avoid bounds on large extra dimensions coming from light KK modes while having TeV scale quantum gravity. The geometry considered here postpone the appearance of KK modes till very short distance (high energy) and all the modes appear at the same time at very high energies.

2.5 4 Fermi interactions

Unlike the usual case in which the first KK state appears at $M_{KK} = 1/R$ and we get $1/M_{KK}^2$ after integrating out KK states, here the KK states are extremely heavy, $M_{KK} = 1/\rho = N/R$. As there appear N such KK states, after integrating out KK states, we get $1/M_{KK}^2 = 1/(NR^2)$ which is suppressed by N. There would be many interesting phenomenology associated with it.

2.6 Warped extra dimension

It would be interesting to see what happens in the warped extra dimensions. We can analyze the spectrum of multi-throat configuration in a similar way, but the result is not as interesting as in flat space. There is a single zero mode whose wave function is all over the extra dimension. Then the excited states appear with wave functions localized near the throats (especially when the curvature is large which is distintively different from flat extra dimensions). It is clearly seen in deconstruction setup [20, 21]. Gauge theory in a warped background has a nontrivial warp factor in front of $\eta^{\mu\nu}F_{\mu 5}F_{\nu 5}$ and it can be deconstructed with a position dependent link VEV $\langle\Phi_i\rangle = \langle\Phi_0\rangle\epsilon^i$ where ϵ corresponds to $e^{-k/\Lambda}$ with k the AdS_5 curvature and Λ the cutoff of the theory with $\epsilon \ll 1$ for highly curved AdS_5 [20]. The mass matrix for N sites is

$$M^2 = \frac{\epsilon^2}{a^2}\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & 1+\epsilon^2 & -\epsilon^2 & 0 & 0 & \cdots & 0 \\ 0 & -\epsilon^2 & \epsilon^2+\epsilon^4 & -\epsilon^4 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 0 & -\epsilon^{2(N-3)} & \epsilon^{2(N-3)}+\epsilon^{2(N-2)} & -\epsilon^{2(N-2)} \\ 0 & \cdots & 0 & 0 & 0 & -\epsilon^{2(N-2)} & \epsilon^{2(N-2)} \end{pmatrix} \tag{2.11}$$

The zero mode eigenstate is

$$A_\mu^{(0)} = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} A_{\mu,i}. \tag{2.12}$$

For the excited states, the analysis is extremely simplified when AdS is highly curved, $\epsilon \ll 1$. The higher mode eigenstates are

$$\begin{aligned} A_\mu^{(N-1)} &= \frac{1}{\sqrt{2}}(A_{\mu,1} - A_{\mu,2}), \\ A_\mu^{(N-2)} &= \frac{1}{\sqrt{6}}(A_{\mu,1} + A_{\mu,2} - 2A_{\mu,3}), \\ &\cdots \\ A_\mu^{(1)} &= \frac{1}{\sqrt{(N-1)N}}(A_{\mu,1} + \cdots + A_{\mu,N-1} - (N-1)A_{\mu,N}), \end{aligned}$$

where the coefficients are determined up to $\mathcal{O}(\epsilon^2)$. $A_\mu^{(N-j)}$ has the eigenvalue of order $\mathcal{O}(\langle\Phi_0\rangle\epsilon^j)$ for $j = 1, \cdots, N-1$ and the 5D interpretation is clear. For higher modes ($m_n \sim \langle\Phi_0\rangle$), the wave function is localized near the UV brane. The lightest mode is mostly localized near the IR brane.

JHEP01(2006)090

The same analysis can be done for the multi-throat configuration which has several IR branes and one UV brane with an Octopus shape. For simplicity, let us consider two IR branes connected to the UV brane. The mass matrix is then

$$M^2 = \frac{\epsilon^2}{a^2} \begin{pmatrix} \epsilon^{2(N-2)} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 1+\epsilon^2 & -1 & 0 & \cdots & 0 \\ 0 & \cdots & -1 & 2 & -1 & \cdots & 0 \\ 0 & \cdots & 0 & -1 & 1+\epsilon^2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & \epsilon^{2(N-2)} \end{pmatrix} \tag{2.13}$$

We can get the eigenstates from the simple UV-IR case. We have 2N-1 sites and there are 2N-1 eigenstates. The zero mode is flat along the extra dimension which is the same as before. The remaining 2N-2 modes are obtained simply by considering even and odd combinations of two N-1 modes. For instance, the lightest modes except the zero mode are

$$\begin{aligned} A_\mu^{(1+)} &= \frac{1}{\sqrt{2(N^2-N+3)}}(-(N-1)A_{\mu,1} + \cdots + A_{\mu,N-1} + 2A_{\mu,N} \\ &\quad +A_{\mu,N+1} + \cdots - (N-1)A_{\mu,2N-1}), \end{aligned} \tag{2.14}$$

$$\begin{aligned} A_\mu^{(1-)} &= \frac{1}{\sqrt{2(N^2-N-1)}}(-(N-1)A_{\mu,1} + \cdots + A_{\mu,N-1} \\ &\quad -A_{\mu,N+1} + \cdots + (N-1)A_{\mu,2N-1}), \end{aligned} \tag{2.15}$$

up to $\mathcal{O}(\epsilon^2)$, and the corresponding eigenvalues are degenerate (twofold degeneracy)

$$m_n = g\langle\Phi_0\rangle\epsilon^{(N-1)}$$

up to $\mathcal{O}(\epsilon^2)$. More precisely the degeneracy is lifted by $1/N$ correction. All the higher modes are similarly obtained and only for the heaviest one, the eigenvalues are $m_n = g\langle\Phi_0\rangle\epsilon$ and $m_n = \sqrt{3}g\langle\Phi_0\rangle\epsilon$ up to $\mathcal{O}(\epsilon^2)$.

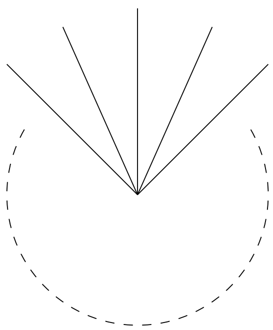
Therefore, the presence of the extra throat does not affect the spectrum of lighter KK states. Only when the KK mass is larger or comparable to the curvature scale, the wave function connects different throats and we get similar results as in flat extra dimensions. This can be easily understood from AdS/CFT correspondence [22−24]. Each throat corresponds to a strongly coupled CFT and each CFT has many resonances (KK modes). The resonances in one CFT is nothing to do with the ones in the other CFT. Thus KK spectrum in AdS which corresponds to the resonances of CFT should not be affected by the presence of other throats.

3. Field theory analysis

It is fairly simple to do the field theory analysis. As the analysis is independent of Lorentz index, let us consider a massless scalar field ϕ in 5 dimensions. The same result will be obtained for massless vector fields, massless gravitons and massless fermions.

JHEP01(2006)090

3.1 Octopus with N legs



First of all, we consider a joint of N intervals at a single point. The figure shows a schematic configuration and dots represent omitted N-5 intervals. The figure just shows the extra dimension and the relative angle between two intervals or the ordering of different intervals do not have any physical meaning in the configuration as there is no space at all beyond the extra dimension denoted by lines in the figure. The lagrangian for a massless scalar field is

$$\mathcal{L} = \int d^4x \left(\int_0^{2\pi\rho} dx_5^{(1)} + \int_0^{2\pi\rho} dx_5^{(2)} + \cdots + \int_0^{2\pi\rho} dx_5^{(N)} \right) \left[\frac{1}{2} \partial_M \phi(x, x_5) \partial^M \phi(x, x_5) \right], \tag{3.1}$$

where $M = 0, 1, 2, 3, 5$ is 5 dimensional Lorentz index. For the octopus of N legs with Neumann boundary conditions at N ends of the legs (for simplicity, we assume all the legs are equal in length, $\pi\rho$),

$$\frac{\partial}{\partial x_5^{(i)}} \phi^{(i)}(x_\mu, x_5^{(i)} = 0) = 0, \tag{3.2}$$

at $x_5^{(i)} = 0$ with $i = 1, \cdots, N$. We restrict our analysis to the case when there is no localized term at the junction. The remaining boundary conditions are i) the wave function should be continuous (as we do not have any extra terms located at special points) and ii) the derivatives should cancel. The first and the second conditions are

$$\phi^{(i)}(x_\mu, x_5^{(i)} = \pi\rho) = \phi^{(j)}(x_\mu, x_5^{(j)} = \pi\rho), \tag{3.3}$$

$$\sum_{i=1}^{N} \frac{\partial}{\partial x_5^{(i)}} \phi^{(i)}(x_\mu, x_5^{(i)} = \pi\rho) = 0. \tag{3.4}$$

Here we introduce coordinates $x_5^{(i)}$ $(i = 1, \cdots, N)$ which runs from 0 (the end of the ith leg) to $\pi\rho$ (the center/junction). We are ready to find the spectrum. Let

$$\phi^{(i)}(x_\mu, x_5^{(i)}) = \sum_n A^{(i)} \phi_n^{(i)} \cos(k_n x_5^{(i)}). \tag{3.5}$$

JHEP01(2006)090

The boundary condition at the ends of the legs are satisfied. The remaining boundary conditions are $N-1$ conditions for the wave functions at the junction and one condition for the cancellation of derivatives at the junction.

As the junction is located at $x_5^{(i)} = \pi\rho$ for all i (equal distance away from the ends), the boundary condition is

$$A^{(i)} \cos(k_n^{(i)} \pi\rho) \;=\; A^{(j)} \cos(k_n^{(j)} \pi\rho). \tag{3.6}$$

which can be satisfied either for i) $k_n^{(i)} \pi\rho = (n^{(i)} + \frac{1}{2})\pi$ or ii) $A^{(i)} = A^{(j)}$ for all $i \neq j$. For i), the final boundary condition is

$$\sum_i A^{(i)} (-1)^{n^{(i)}+1} \;=\; 0. \tag{3.7}$$

For ii), the condition is

$$\left(\sum_i A^{(i)} \right) \sin(k_n^{(i)} \pi\rho) \;=\; N A^{(1)} \sin(k_n^{(i)} \pi\rho), \tag{3.8}$$

and it can be satisfied only when $k_n^{(i)} \pi\rho = n^{(i)}\pi$ since $A^{(1)} \neq 0$.

Now all the eigenvalues are determined. Let us consider how many degenerate states are there for each k_n. For i), we have $N-1$ independent solutions which can be written in terms of a N dimensional vector v

$$v \;=\; (A^{(1)}, A^{(2)}, \cdots, A^{(N)}). \tag{3.9}$$

as

$$\begin{aligned} v \;=\; & \frac{1}{\sqrt{2}} \sqrt{\frac{2}{\pi\rho}} (1, -1, 0, \cdots, 0) \\ & \frac{1}{\sqrt{6}} \sqrt{\frac{2}{\pi\rho}} (1, 1, -2, \cdots, 0) \\ & \cdots \\ & \frac{1}{\sqrt{(N-1)N}} \sqrt{\frac{2}{\pi\rho}} (1, 1, 1, \cdots, -(N-1)) \end{aligned}$$

For ii), all the coefficients are determined and there is a single state.

$$v \;=\; \frac{1}{\sqrt{N}} \sqrt{\frac{2}{2^{\delta_{n,0}} \pi\rho}} (1, 1, 1, \cdots, 1)$$

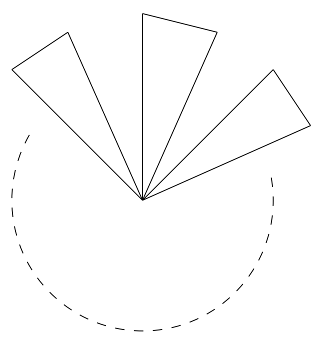
We should be careful here. For ii), we can imagine a wave function which is connected with different $n^{(i)}$s at differnt $x_5^{(i)}$. As we know that there is a zero mode with a flat potential, we can check whether the arbitrary $n^{(i)}$ can yield the orthogonality condition. For $n^{(i)} \neq n^{(j)}$, the wave functions are orthogonal at the ith leg. The lightest mode (except the zero mode) should not include $n^{(i)} = 0$ as they will generate a nonzero positive contribution when we consider orthogonality condition with the zero mode. $n^{(i)} \geq 1$ is required from the consideration and the lightest mode is $n^{(i)} = 1$ for all i. Similar reasoning gives $n^{(i)} = 2$ and higher and we can simply replace $n^{(i)} = n$.

Now the spectrum is alternating. We have a single mode at $M_n = n/\rho$ and $N-1$ modes in between nth and $n+1$th mode ($M_n = (n+\frac{1}{2})/\rho)$).

Asymmetry between the degeneracy of n/ρ and $(n+\frac{1}{2})/\rho$ modes can be understood as follows. We put Neumann boundary conditions at the ends of the legs and thus the states with Dirichlet boundary conditions are projected out.

If we impose Dirichlet boundary condition at the ends of the legs, we would encounter the opposite case. There is no zero mode and a single mode at $M_n = (n+\frac{1}{2})/\rho$ and $N-1$ modes at $M_n = (n+1)/\rho$ with $n \geq 0$.

3.2 Flower with N leaves



To see the picture clearly, let us consider a flower configuration where N rings are attached at the same point (center). Each ring has a circumference $2\pi\rho$. We can do the similar analysis. Now $x_5^{(i)}$ is from 0 to $2\pi\rho$ and

$$\phi^{(i)}(x_\mu, x_5^{(i)}) = \sum_n (A^{(i)}\phi_n^{(i)} \cos(k_n x_5^{(i)}) + B^{(i)}\phi_n^{(i)} \sin(k_n x_5^{(i)})). \tag{3.10}$$

For each ring(leaf), the boundary condition corresponding to the end points of Octopus is

$$\phi^{(i)}(x_5^{(i)} + 2\pi\rho) = \phi^{(i)}(x_5^{(i)}), \tag{3.11}$$

and it determines $k_n^{(i)} 2\pi\rho = 2\pi n^{(i)}$ and $k_n^{(i)} = n^{(i)}/\rho$. The remaining boundary condition at the center is the same. If we assign the center to be $x_5^{(i)} = \pi\rho$, the first $N-1$ boundary condition requires

$$A^{(i)}(-1)^{n^{(i)}+1} = A^{(j)}(-1)^{n^{(j)}+1}. \tag{3.12}$$

The special limit is when all $A^{(i)} = 0$. The second boundary condition is automatically satisfied. For each $\phi^{(i)}$, there are incoming and outgoing derivatives which cancel with each other. Therefore, for each $n^{(i)}$, we can have $N+1$ independent solutions except when $n^{(i)}=0$. For $n^{(i)}=0$ for all is, we have the usual zero mode.

$$\begin{aligned} v &= \frac{1}{\sqrt{N}}\frac{1}{\sqrt{2\pi\rho}}(1,1,1,\cdots,1) \\ w &= (0,0,0,\cdots,0) \end{aligned}$$

Note that you do not need to have the same $k_n^{(i)}$ for different is. The lightest mode appears when all $k_n^{(i)}=1$. There are $N+1$ such states which are degenerate with $k_n=1/\rho$. One is

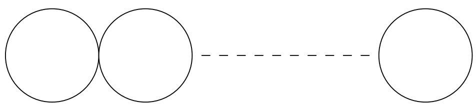
$$\begin{aligned} v &= \frac{1}{\sqrt{N}}\frac{1}{\sqrt{\pi\rho}}(1,1,1,\cdots,1) \\ w &= (0,0,0,\cdots,0) \end{aligned}$$

and the other N states are

$$\begin{aligned} v &= (0,0,0,\cdots,0) \\ w &= \frac{1}{\sqrt{\pi\rho}}(1,0,0,\cdots,0) \\ &\quad \frac{1}{\sqrt{\pi\rho}}(0,1,0,\cdots,0) \\ &\quad \cdots \\ &\quad \frac{1}{\sqrt{\pi\rho}}(0,0,0,\cdots,1) \end{aligned} \tag{3.13}$$

For the latter case, it can be thought that the modes will be lighter than n/ρ as there is only one ring that gives Kaluza-Klein mass. However, there is no wave function outside of the ring and the result is the same as the case with a single ring with a radius ρ. You can see that there are $N+1$ states at each n/ρ except $n=0$ (a single zero mode).

3.3 Caterpillar



Finally let us consider a ring that is attached with each other but the ring intersects only with two nearest neighbor rings (except the edge ring which intersects with only one ring). It would be a sequence of shape 8 and let us call it 'caterpillar'. From the boundary conditions

$$\phi^{(i)}(x_5^{(i)}+2\pi\rho) = \phi^{(i)}(x_5^{(i)}), \tag{3.14}$$

we can determine $k_n^{(i)} = n^{(i)}/\rho$. When $A^{(1)} \neq 0$, the wave function is continuous if

$$A^{(i)}(-1)^{n^{(i)}} = A^{(i+1)}. \tag{3.15}$$

There is no condition for $B^{(i)}$ as the derivatives cancel within the same ring. The situation is the same as in the flower configuration. The first one for $n = 0$ is

$$\begin{aligned} v &= \frac{1}{\sqrt{N}} \frac{1}{\sqrt{2\pi\rho}}(1, 1, 1, \cdots, 1) \\ w &= (0, 0, 0, \cdots, 0) \end{aligned}$$

and for $n^{(i)} \neq 0$,

$$\begin{aligned} v &= \frac{1}{\sqrt{N}} \frac{1}{\sqrt{\pi\rho}}(1, (-1)^{n^{(i)}}, 1, \cdots, (-1)^{(n^{(i)}N)}) \\ w &= (0, 0, 0, \cdots, 0) \end{aligned}$$

and the other N states are

$$\begin{aligned} v &= (0, 0, 0, \cdots, 0) \\ w &= \frac{1}{\sqrt{\pi\rho}}(1, 0, 0, \cdots, 0) \\ &\quad \frac{1}{\sqrt{\pi\rho}}(0, 1, 0, \cdots, 0) \\ &\quad \cdots \\ &\quad \frac{1}{\sqrt{\pi\rho}}(0, 0, 0, \cdots, 1) \end{aligned} \tag{3.16}$$

There are totally $N + 1$ degenerate states for each $k_n = n/\rho$.

However, there appears much lighter states in this case. Suppose $N = 2k+1$. Then we can imagine a configuration in which $k_n^{(i)} = 0$ for all is except $i = k+1$ and $k_n^{(k+1)} = 1/\rho$. The wave function is

$$\begin{aligned} v &= \frac{1}{\sqrt{N}} \frac{1}{\sqrt{2^{\delta_{n^{(i)},0}}\pi\rho}}(1, 1, 1, \cdots, 1, -1, \cdots, -1, -1, -1) \\ w &= (0, 0, 0, \cdots, 0) \end{aligned}$$

Now as there is an $\frac{1}{\sqrt{N}}$ volume suppression in the wave function and the mode is still orthogonal to the zero mode. The contributions of the first k rings cancel the ones of the last k rings and $k+1$ ring wave function is orthogonal since $n^{(k)} = 0$ for the zero mode and $n^{(k)} = 1$ for the mode considered here. The Kaluza-Klein mass only comes from a single ring and we get $k^{4D} = 1/(\sqrt{N}\rho)$ rather than $1/\rho$. Here the configuration is uniquely determined since the change of the wave function ($n^{(i)} \neq 0$) should be located in the middle to balance the wave function such that it can be orthogonal to the zero mode.

If we consider $n^{(i)} = 1$ for two is, we can not make the wave function to be orthogonal to the zero mode if $N = 2k+1$. Instead, we can consider $N = 2k$. As there are $2k-2$ rings with $n^{(i)} = 0$, they should be evenly divided into positive and negative amplitudes. It is possible when the first nonzero $n^{(i)}$ and the second nonzero $n^{(i)}$ has a separation of $k-1$. There are k such possibilities. Although it would be interesting to study the spectrum of these cases in detail, we will not pursue it here.

You can see the huge difference between the flower and the caterpillar configurations. There is no constraint from the derivative matching and the momentum in one ring can be different from the one in the other ring in principle. As a consequence the lightest mode start to appear at $1/(\sqrt{N}\rho)$ although the actual configuration is not a homogeneous variation along $2\pi\rho$ but a rapid variation only at a local region. This is in accord with the deconstruction result of two centered Octopus.

We stress here that it is the presence of a junction from which all the subsegments or rings are connected and they make it possible to raise the scale of the Kaluza-Klein excitations.

4. Conclusion

In this paper we have shown that the spectrum of Kaluza-Klein particles can be rich and interesting even with a single extra dimension. Depending on how the extra dimension is connected with each other, the KK spectrum appears entirely differently. The most interesting aspect is that we can defer the appearance of the lightest KK modes as high as we want. This is impossible with a simple circle compactification or an orbifolding of it. With a single extra dimension the lightest KK mode is directly linked to the size of the extra dimension (1/R) if the extra dimension is a simple circle or an interval. Several examples considered in this paper shows that the relation no longer holds if several extra dimension is connected with a common point, so called 'junction' or 'center'. This enables us to have TeV scale quantum gravity with a single extra dimension and a lightest KK mass of 100 MeV.

The fact that this new setup is just 5 dimensional spacetime is important. This opens an entirely new era for figuring out what would be the shape of the extra dimension relevant to the real world. Before going to higher dimensional theory, we can study a lot of examples with a single extra dimension. Orbifold GUT in 6D [25 – 29] has more freedom over 5D model since we can use two orbifolding parity and two Wilson lines. However, now with the setup considered here, we can do the similar thing in 5D by attaching two or three intervals. Model building should be seriously done with these new setups. It would be possible to build a simple model in 5D. The configuration might be regarded as ad hoc. However, it can be understood as an effective description of the underlying theory and most of important physics questions can be addressed without relying on what the exact underlying theory is.

Although we studied the single extra dimension only, the 'center' or the 'junction' can play the same role when two or more spatial extra dimensions are attached. Also the 'center' or the 'junction' can be generalized to arbitrary higher dimensions. Furthermore, we have not introduced any local kinetic terms or mass terms here but we can study the general cases in which the 'center' has a special interactions. Boundary conditions at the leg also can be generalized. We leave many detailed example studies for the future work. Phenomenological constraints on the extra dimension also should be restated after considering several variations of the simple compactification.

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